A diagrammatic derivation of the meson effective masses in the neutral color-flavor-locked phase of Quantum Chromodynamics

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We offer a diagrammatic derivation of the effective masses of the axial flavor excitations in the electrical and color neutral CFL phase of QCD. In particular we concentrate on the excitations with the quantum numbers of the kaons: we show how their effective chemical potentials, responsible of their Bose-Einstein condensation and found previously on the basis of pure symmetry arguments, arise at the microscopic level by loop effects. We perform also the numerical evaluation of the relevant loops in the whole CFL regime $M_s^2/2\mu\Delta \leqslant 1$, showing the existence of the enhancement of the kaon condensation with respect to the lowest order result. Finally we discuss the role of color neutrality in the microscopic calculation.

I. INTRODUCTION

The study of the phase diagram of Quantum Chromodynamics (QCD) has attracted a lot of interest in the last years. In particular, the major theoretical improvement of the knowledge of the high density and low temperature regime of QCD has been achieved by phenomenological studies based both on Nambu-Jona Lasinio (NJL) models [1, 2, 3] and abstract formulations relying on Ginzburg-Landau expansions of the free energy [4, 5]. In these extreme conditions the QCD ground state is likely to be a color superconductor (for reviews see [6, 7, 8, 9, 10, 11, 12, 13, 14, 15]).

At asymptotic densities (or equivalently, at asymptotic baryon chemical potential μ) and low temperatures the ground state of three flavor quark matter is the color-flavor-locked state (CFL) [16, 17], where the color and flavor symmetries of QCD are broken spontaneously by a di-quark vacuum expectation value proportional to the gap parameter Δ, but a residual group linking color and vector flavor symmetries is left. In the CFL phase all the quarks pair among themselves and get an effective mass Δ.

When the density is decreased, finite quark masses effects and electrical and color neutrality conditions break the symmetric CFL state into a less symmetric state. Examples include the gapless CFL state [18], kaon condensed states [19, 20], inhomogeneous three flavor color superconductors [21], gapless two flavor color superconductors, both in the homogeneous [22, 23] and inhomogeneous realizations [24], and spin one pairing [25, 26]. In particular, denoting by M_s the in-medium strange quark mass, we remind that the parameter useful to describe the CFL→gCFL transition is $M_s^2/2\mu\Delta$: if it is smaller than 1 then the ground state is the CFL phase; on the other hand, if $M_s^2/2\mu\Delta > 1$ then the ground state is the gapless gCFL phase, characterized by gapless quark excitations in the spectrum. Unfortunately gapless homogeneous phases suffer the problem of Meissner antiscreening: the squared Meissner masses of some of the gluons, when evaluated in the one loop approximation, turn out to be negative, both in the two flavor [27] and in the three flavor [28] case. This fact is interpreted as an instability toward a gluon condensed state [29] and/or an inhomogeneous ground states, with meson currents spontaneously generated [30, 31, 32] and non isotropic pairing [33] among quarks.

In the CFL state the chiral symmetry is broken, and an octet of pseudoscalar meson fields appear in the spectrum as required by the Goldstone theorem. Beside them two flavor singlet fields arise because of the breaking of the baryon $U(1)_V$ and of the axial $U(1)_A$ which is restored at high densities.

The fact that all the quarks are gapped implies the existence of two characteristic energy scales in the CFL phase: the gap parameter Δ, that roughly speaking measures the mass of the fermion excitations, and the high baryon chemical potential $\mu \gg \Delta$. Since the gap is much smaller than the chemical potential (and thus of the Fermi momenta of the quarks) the quark dynamics is dominated by the fermion modes living in a thin shell around the Fermi surface: one can therefore write a renormalized effective fermion action (in the sense of Wilson), in terms of the soft modes that live near the Fermi surface [8, 35, 36]. Moreover, since the fermion excitations are gapped, for energies much smaller than the gap one can describe the low energy physics of the CFL phase by means of the light flavor excitations [19, 37].

It was noticed [19] that a finite strange quark mass can modify the CFL ground state before the transition to gCFL occurs: as a matter of fact, in the limit of isospin symmetry $M_u = M_d$ (capital letters denote in-medium quark masses), the excitations with kaon quantum numbers feel an effective chemical potential that can become larger than the kaon mass if the strange quark mass is large enough. If this is the case then it is possible to lower the free energy of the system by forming a kaon Bose-Einstein condensate. Astrophysical implications of the presence of a kaon condensed phase in the core of a compact star have been widely studied [38]. It becomes

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clear then that the knowledge of the effective masses of the kaon modes in the CFL phase is important for the determination of the true ground state of three flavor QCD.

The important project of writing an effective lagrangian for the flavor excitations, and thus of their effective masses, in the CFL phase of QCD is a classical topic widely covered [39, 40, 41, 42, 43]; it has been considered more recently in [44, 45, 46, 47], where the effects of electrical and color neutrality and of the finite quark masses have been introduced. However, we have noticed that up to now there exists no clear derivation of the effective masses of the kaons in the CFL phase, and in particular of their effective chemical potentials, based on a purely microscopic model of quarks interacting with the flavor excitations. We wish to cover this important topic here.

The main scope of our work is to show how the effective chemical potentials felt by the kaon fields, responsible of the kaon condensation phenomenon and deduced on the basis of symmetries in [19], arise as loop effects once the octet of the $SU(3)_A$ flavor excitations is introduced as an external field [40, 48]. Recently the same procedure has been applied in Ref. [49] to the Goldstone modes of the inhomogeneous three flavor superconductive state.

From the very beginning we work in an electrically and color neutral state. This is the case because any homogeneous ground states in QCD has to be color neutral [44, 46, 50]. Moreover, one can show, by using a low energy effective lagrangian of the CFL phase [37, 44, 46] that color neutrality implies the decoupling, from the low energy spectrum, of the colored degrees of freedom, leaving a lagrangian written in terms of the only color singlet axial flavor excitations (we neglect the flavor singlets corresponding to $U(1)_B$ and $U(1)_A$). This turns to be equivalent to impose color neutrality in the microscopic model [44, 46]: as a consequence, in order to reproduce properly such a low energy effective lagrangian by a microscopic theory of interacting quarks, the starting point must be a color neutral state.

We perform the calculation of the effective meson chemical potentials both analytically, by an expansion in powers of $M_s^2/\mu\Delta$, and numerically. The method presented here is applicable to the gapless CFL phase as well. The analytical calculation shows an interesting aspect: if one does not forces color neutrality, and leaves the color chemical potentials arbitrary, then one notices that the effective mass of the kaons does not recover the value obtained by the effective theory. Only if the color chemical potentials are chosen in order to fulfill the neutrality condition, that is only if one considers a neutral state, then the two results match.

The paper is organized as follows: in Section II we describe the quark content of the model. In Section III we introduce the $SU(3)_A$ flavor excitations as external fields via the Eguchi recipe [48], defining their coupling to the quarks and their one loop effective Lagrangian by integrating over the quarks in the functional integral. Section IV is the main body of the paper: we compute both analytically and numerically the effective chemical potentials felt by the flavored meson fields, once the chemical potentials of the quarks are chosen in order to fulfill the neutrality conditions (since we work in the limit $M_u = M_d = 0$ only the kaons are massive, while all of the other meson modes are massless [19, 41, 42]). Finally, we draw our conclusions, indicating some consequences of our results and possible improvements of the work.

II. THE EFFECTIVE QUARK LAGRANGIAN

In this paper we deal with three flavor quark matter, whose interaction is modeled by a local Nambu-Jona Lasinio (NJL) lagrangian [51] (for reviews see [12, 52, 53]). The main limitation of NJL models is the lack of gluons; nevertheless, as it is consistent with the global symmetries of QCD, it is believed to be able to capture the essential physics of the problem.

At finite chemical potential and in presence of color condensation the quark lagrangian is given by

$$\mathcal{L} = \bar{\psi}\left(i\partial_\mu\gamma^\mu + \hat{\mu}\gamma_0\right)\psi - M_f\bar{\psi}_f\psi_f + \mathcal{L}_\Delta \ . \qquad (1)$$

In the above equation $\hat{\mu}$ is the quark chemical potential matrix, with color and flavor indices. It depends on μ (the average quark chemical potential), μ_e (the electron chemical potential), and μ_3, μ_8 (color chemical potentials) [18]. For color and electric neutrality to be implemented it is sufficient to consider only these chemical potentials, related as they are to the charge matrix and the diagonal color operators $T_3 = \frac{1}{2}\mathrm{diag}(1,-1,0)$ and $T_8 = \frac{1}{2\sqrt{3}}\mathrm{diag}(1,1,-2)$ (in general one should introduce a color chemical potential for each $SU(3)$ color charge; however, as shown in [54], for the condensate with the color-flavor structure considered in this paper it is enough to consider only μ_3 and μ_8, since the charges related to the other color generators automatically vanish). Therefore the matrix $\hat{\mu}$ is written as follows

$$\hat{\mu}_{ij}^{\alpha\beta} = \left(\mu\delta_{ij} - \mu_e Q_{ij}\right)\delta^{\alpha\beta} + \delta_{ij}\left(\mu_3 T_3^{\alpha\beta} + \frac{2}{\sqrt{3}}\mu_8 T_8^{\alpha\beta}\right) \qquad (2)$$

with $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ ($i,j = 1,3$ flavor indices; $\alpha,\beta = 1,3$ colour indices).

The term $\mathcal{L}_\Delta$ is responsible for color condensation, and is given in the mean field approximation by

$$\mathcal{L}_\Delta = -\frac{1}{2}\sum_{I=1}^{3}\left(\Delta_I(\boldsymbol{r})\psi^\dagger_{\alpha i}\gamma_5\epsilon^{\alpha\beta I}\epsilon_{ijI}C\psi^*_{\beta j} + h.c.\right) \ . \qquad (3)$$

Eq. (3) describes the fact that in the ground state one has a non-vanishing expectation value of the di-quark field operator

$$\langle\psi(\boldsymbol{r})_{\alpha i}\psi(\boldsymbol{r})_{\beta j}\rangle \propto \Delta_I(\boldsymbol{r})\epsilon_{\alpha\beta I}\epsilon_{ijI} \neq 0 \ . \qquad (4)$$

In this work we are interested to the CFL phase [16, 18]: in this case $\Delta_1 = \Delta_2 = \Delta_3 \equiv \Delta$, independent of the space coordinates $\boldsymbol{r}$.

Finally, M_f in Eq. (1) denote the in-medium quark mass of the flavor f. Here we treat the quark masses at the leading order in M_f^2/μ: at this order the effect of the finite mass is a shift of the quark chemical potentials μ_f by the amount $-M_f^2/2\mu$. This is a widely used approximation, which captures the main role of the finite quark masses, namely the reduction of their Fermi spheres (in addition to that, the approximation allows for easier calculations).

In this paper we adopt the high density effective description of QCD [8, 35, 36]: this approximation amounts to consider only the quarks with momenta close to the Fermi surface; this approximation is justified since in the weak coupling regime, to which we are interested here, the quarks living inside the Fermi sphere are Pauli blocked and are not relevant for the dynamics; moreover, the negative energy fields give rise to operators that are formally suppressed by inverse powers of μ and therefore give a negligible contribution to the quark propagator.

The high density effective lagrangian of the quarks in the CFL phase of QCD has been discussed many times in the literature (see for example [8, 41, 42]), therefore here we simply quote the result in the momentum space, namely

$$\mathcal{L} = \frac{1}{2}\int \frac{d\boldsymbol{n}}{4\pi}\, \chi_A^\dagger \begin{pmatrix} K_{AB}(\ell) & -\Delta_{AB} \\ -\Delta^\star_{AB} & \tilde{K}_{AB}(\ell) \end{pmatrix} \chi_B + L \to R \ . \tag{5}$$

Here $A = 1, \ldots, 9$ is a color-flavor index; the rotation to the new basis is performed by means of the matrices F_A defined in [28]. The kinetic terms are defined as $K_{AB} = V\cdot\ell\ \delta_{AB} + \delta\mu_{AB}$, $\tilde{K}_{AB} = \tilde{V}\cdot\ell\ \delta_{AB} - \delta\mu_{AB}$. The quark momenta are measured as $\boldsymbol{p} = \mu\boldsymbol{n} + \boldsymbol{\ell}$, $p_0 = \ell_0$, with $\boldsymbol{n}$ a unit vector denoting the Fermi velocity of the quarks and μ is a reference large momentum, usually equal to the baryon chemical potential. The chemical potential of the quark with index A is written as $\mu_A = \mu + \delta\mu_A$ and $\delta\mu_{AB} \equiv \delta\mu_A\delta_{AB}$. The entry $\delta\mu_A$ contains also the effective mass $-M_f^2/2\mu$ of the flavor f. The gap matrix is given by $\Delta_{AB} = \Delta_I(\boldsymbol{r})\mathrm{Tr}[\epsilon_I F_A^T \epsilon_I F_B]$. Finally, we have introduced the Nambu-Gorkov doublet for the left-handed fields,

$$\chi = \begin{pmatrix} \psi(\boldsymbol{n}) \\ C\psi^*(-\boldsymbol{n}) \end{pmatrix} \ . \tag{6}$$

The calculations presented in the Section IV are devoted to the evaluation of the masses of the excitations related to the breaking of the $SU(3)_A$ symmetry. In that context one needs to add mass corrections to the high density effective theory (HDET). One way to do that consistently in a NJL model is to introduce an anti-gap term to the above lagrangian: it has been treated many times in the literature, and its role in the calculation of pseudo-Nambu-Goldstone modes in NJL studies of high density QCD has been emphasized. In particular we need the lagrangian which describes antiquark–antiquark pairing, introduced in the HDET formalims in [41, 55]. On the other hand, starting from the QCD quark-gluon vertex it is possible to get effective four fermion interactions by integrating out the electric gluons, see for example the clear discussion in [42]. The introduction of such vertices is essential in QCD since it has been shown that the anti-gap lagrangian gives rise to gauge dependent values of the shift in the vacuum energy generated by the finite values of the quark masses [42]; also, when one considers quark-antiquark pairing, the shift of the vacuum energy sum up to zero and one is left with the quark-quark contribution only, see Eq. (39) of Ref. [42]. In any case (NJL or QCD) the contribution of this kind of corrections to the squared meson masses is highly suppressed since it contains light quark mass insertions. As a consequence one can obtain them in the microscopic calculation by putting $M_s = 0$ in the quark propagators, and the well known results hold [39, 40, 41, 42, 43].

We close this section by defining the left handed quark propagator in momentum space,

$$S(\ell)^{-1}_{AB} = \begin{pmatrix} K_{AB}(\ell) & -\Delta_{AB} \\ -\Delta^\star_{AB} & \tilde{K}_{AB}(\ell) \end{pmatrix} \ ; \tag{7}$$

the analogous Green function of the right handed quarks is obtained trivially from the previous one. From now on we consider the limit $M_u = M_d = 0$. Thus the electrical and color neutrality conditions are fulfilled by [18, 56]

$$\mu_e = \mu_3 = 0 \ , \tag{8}$$

$$\mu_8 \approx -\frac{M_s^2}{2\mu} \ . \tag{9}$$

After these assumptions are made we write explicitly the chemical potentials entering into the quark Lagrangian (5) in the case of the neutral CFL phase: $\mu_A = \mu + \delta\mu_A$ with [18]

$$\delta\mu_1 = \delta\mu_{ur} = \mu - \frac{M_s^2}{6\mu} \ , \tag{10}$$

$$\delta\mu_2 = \delta\mu_{dg} = \delta\mu_1 \ , \tag{11}$$

$$\delta\mu_3 = \delta\mu_{bs} = \delta\mu_1 \ , \tag{12}$$

$$\delta\mu_4 = \delta\mu_{dr} = \delta\mu_1 \ , \tag{13}$$

$$\delta\mu_5 = \delta\mu_{ug} = \delta\mu_1 \ , \tag{14}$$

$$\delta\mu_6 = \delta\mu_{sr} = \delta\mu_1 - \frac{M_s^2}{2\mu} \ , \tag{15}$$

$$\delta\mu_7 = \delta\mu_{ub} = \delta\mu_1 + \frac{M_s^2}{2\mu} \ , \tag{16}$$

$$\delta\mu_8 = \delta\mu_{sg} = \delta\mu_1 - \frac{M_s^2}{2\mu} \ , \tag{17}$$

$$\delta\mu_9 = \delta\mu_{db} = \delta\mu_1 + \frac{M_s^2}{2\mu} \ . \tag{18}$$

From the above equations we notice that the chemical potentials of the quarks with $A = 1, \ldots, 5$ is the same: thus it results more convenient, in the HDET momenta decomposition, to choose $\mu + \delta\mu_1$ as the large reference momentum.

III. COUPLING OF THE $SU(3)_A$ GOLDSTONE BOSONS TO THE QUARKS

In this section we derive the coupling of the quarks to the Goldstones. Following Ref. [48] the $SU(3)_A$ excitations are introduced in the model by rotating the CFL quark condensate. This rotation can be achieved on the left handed fields by means of the axial flavor transformations defined by [40]

$$\psi_{\alpha i} \to \psi_{\alpha k} \left(\mathcal{U}^\dagger\right)_{ki} \ , \quad \mathcal{U} \equiv \exp\left\{ i \frac{\pi_a \lambda_a}{2F} \right\} , \tag{19}$$

where $a = 1, ..., 8$, λ_a are the Gell-Mann matrices, normalized as $\mathrm{Tr}\{\lambda_a \lambda_b\} = 2\delta_{ab}$, and F is the decay constant. For the right fields the transformation is analogous.

In order to properly describe the $SU(3)_A$ Goldstone excitations we promote the quark mass matrix M to a spurion field that has definite transformations under chiral transformations, that is

$$M \to L M R^\dagger \ . \tag{20}$$

An analogous transformation is introduced for the charge matrix: $Q \to L Q L^\dagger$. In this way the effective quark mass term for the left handed fields, namely $-\psi^\dagger (M M^\dagger / 2\mu)\psi$, and the charge chemical potential term $-\psi^\dagger(\mu_e Q)\psi$ are invariant under the chiral transformations (the same is true for the right handed fields), and thus under the quark rotation defined by Eq. (19). The lagrangian is thus given by

$$\mathcal{L} = \int \frac{d\boldsymbol{n}}{8\pi} \chi_A^\dagger \begin{pmatrix} K_{AB}(\ell) & -\Xi^\star_{BA} \\ -\Xi_{AB} & \tilde{K}_{AB}(\ell) \end{pmatrix} \chi_B \ , \tag{21}$$

where

$$\Xi_{AB} = \Delta_I^\star(\boldsymbol{r}) \mathrm{Tr}[\epsilon_I (F_A \mathcal{U}^\dagger)^T \epsilon_I F_B \mathcal{U}^\dagger] \ . \tag{22}$$

After the rotation is done, one sets the chemical potentials and the quark masses to their values in the neutral CFL phase. From the above equation it is clear that the $SU(3)_A$ flavor excitations are introduced only as a rotation of the quark condensate, and disappear from the spectrum if at this stage one sets $\Delta = 0$.

Eq. (21) is the non linear realization of a theory of quarks interacting with an octet of external fields. In order to obtain a kinetic term for the "pions" π_a we linearize the theory by expanding $\mathcal{U}$ in Eq. (19) up to the second order in the π_a fields: this results in three-body and four-body interaction interaction terms,

$$i\mathcal{L}_{\chi\chi\pi} = + \frac{i\,\pi_a}{2F} \int \frac{d\boldsymbol{n}}{8\pi} \chi_A^\dagger (\mathcal{G}_3)^a_{AB}\, \chi_B \ , \tag{23}$$

$$i\mathcal{L}_{\chi\chi\pi\pi} = + \frac{\pi_a \pi_b}{8F^2} \int \frac{d\boldsymbol{n}}{8\pi} \chi_A^\dagger (\mathcal{G}_4)^{ab}_{AB}\, \chi_B \ . \tag{24}$$

The expressions of $\mathcal{G}_3, \mathcal{G}_4$ are as follows:

$$\mathcal{G}_3 = \begin{pmatrix} 0 & -(K^{3a}_{BA})^\star \\ K^{3a}_{AB} & 0 \end{pmatrix} \ , \tag{25}$$

$$\mathcal{G}_4 = \begin{pmatrix} 0 & (K^{4ab}_{BA})^\star \\ K^{4ab}_{AB} & 0 \end{pmatrix} \ . \tag{26}$$

The off-diagonal entries are defined as

$$K^{3a}_{AB} = \Delta_I^\star(\boldsymbol{r})\ \mathrm{Tr}[\epsilon_I \lambda_a^T F_A^T \epsilon_I F_B + \epsilon_I F_A^T \epsilon_I F_B \lambda_a] \ , \tag{27}$$

$$K^{4ab}_{AB} = \Delta_I^\star(\boldsymbol{r})\ \mathrm{Tr}[\epsilon_I \lambda_a^T \lambda_b^T F_A^T \epsilon_I F_B + \epsilon_I F_A^T \epsilon_I F_B \lambda_a \lambda_b + 2\epsilon_I \lambda_a^T F_A^T \epsilon_I F_B \lambda_b] \ . \tag{28}$$

Integrating over the fermion fields in the generating functional of the model one is left with the effective lagrangian in momentum space $\mathcal{L}(p) = \mathcal{L}_{s.e.}(p) + \mathcal{L}_{tad}$ with [8]

$$i\mathcal{L}_{tad} = + \left(\frac{\pi_a \pi_b}{8F^2}\right) \frac{\mu^2}{4\pi^3} \int \frac{d\boldsymbol{n}}{4\pi} \int d^2\ell\ \mathrm{Tr}[S(\ell)\mathcal{G}^4] \ , \tag{29}$$

$$i\mathcal{L}_{s.e.}(p) = -\frac{1}{2} \left(i\frac{\pi_a}{2F}\right) \left(i\frac{\pi_b}{2F}\right) \frac{\mu^2}{4\pi^3} \int \frac{d\boldsymbol{n}}{4\pi} \int d^2\ell\ \mathrm{Tr}[S(\ell+p)\mathcal{G}^3 S(\ell)\mathcal{G}^3] \ ; \tag{30}$$

the overall minus sign in Eq. (30) is due to the fermion loop. We have already kept into account of the $L+R$ contribution, and the trace is intended in Nambu-Gorkov as well as in color-flavor indices. The quark propagator is defined in Eq. (7) with chemical potentials given in Eqs. (10) - (18).

IV. MESON MASSES

The most important task in this work is the computation of the squared masses of the axial flavor excitations, to which we turn. We begin with the analytical calculation at small $M_s^2/2\mu\Delta$; the analytical results are confirmed by a numerical analysis, that allows to make evaluations up to $M_s^2/2\mu\Delta = 1$ (for higher values of the ratio $M_s^2/2\mu\Delta$ one enters the gCFL regime: in this case the role of gapless fermions has to be kept into account, together with $\mu_e \neq 0$ and $\mu_3 \neq 0$).

A. Analytical results

In this section we present analytical results that can easily obtained by expanding the quark propagator in powers of $M_s^2/\mu\Delta$. The result of the calculation is in agreement with the result of [19, 42], obtained by the authors on the basis of pure symmetry arguments. For simplicity, since we are interested to values of $M_s \gg M_{u,d}$, we set $M_{u,d} = 0$ in the quark loops.

To begin with we evaluate analytically the squared masses of the excitations at the leading order in the parameter $M_s^2/\mu\Delta$. The result is achieved evaluating $\mathcal{L}(p=0)$ with $\mathcal{L}$ given by the sum of Eqs. (29) and (30). We find

$$\mathcal{L}(p=0) \;=\; \mu_4^2 K^+ K^- + \mu_6^2 K^0 \bar{K}^0 \;, \tag{31}$$

where the meson fields are defined in terms of the pion fields π_a as usual: $K^\pm = (\pi_4 \mp i\pi_5)/\sqrt{2}$, and $K^0/\bar{K}^0 = (\pi_6 \mp i\pi_7)/\sqrt{2}$. In the above equation

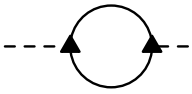
$$\mu_4^2 \;=\; \left(\frac{M_s^2}{2\mu}\right)^2 \;, \tag{32}$$

$$\mu_6^2 \;=\; \mu_4^2 \;. \tag{33}$$

For the other meson excitations we find a vanishing effective mass, $\mathcal{L}(p=0) = 0$.

The results (32), (33) are in agreement with those of Bedaque and Schäfer (BS) usually quoted in the literature [19]. We now discuss how they are obtained in the microscopic calculation. From now on we consider only the charged kaons mode: the neutral ones are treated in a similar way.

In Fig. 1 we draw the one loop effective action of π_4 at zero external momentum (for π_5 one gets the same results). External lines denote the meson field, solid line are fermion propagators. We find



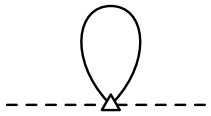
$$= \frac{-i\pi_4^2}{8F^2}\frac{\mu^2}{4\pi^3}\mathcal{K}(M_s) \;, \tag{34}$$

with $\mathcal{K}(M_s) = \mathcal{K}_0 + \mathcal{K}_2$ and

$$\mathcal{K}_0 = 16\Delta^2 \int d^2\ell \frac{-\ell_0^2 + 3\Delta^2 + \ell_\parallel^2}{(\ell_0^2 - \ell_\parallel^2 - \Delta^2)(\ell_0^2 - \ell_\parallel^2 - 4\Delta^2)} \;, \tag{35}$$

$$\begin{aligned}\mathcal{K}_2 \;&=\; -16\Delta^2 \left(\frac{M_s^2}{2\mu}\right)^2 \\ &\times \int d^2\ell \frac{-\ell_0^4 + \ell_0^2\Delta^2 + 3\Delta^4 + 4\Delta^2\ell_\parallel^2 + \ell_\parallel^4}{(\ell_0^2 - \ell_\parallel^2 - \Delta^2)^3(\ell_0^2 - \ell_\parallel^2 - 4\Delta^2)} .\end{aligned} \tag{36}$$

Moreover we find

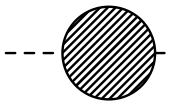


$$= \frac{-i\pi_4^2}{8F^2}\frac{\mu^2}{4\pi^3}\mathcal{N}(M_s) \;, \tag{37}$$

with $\mathcal{N}(M_s) = -\mathcal{K}_0 + \mathcal{N}_2(M_s)$ and

$$\mathcal{N}_2 = 4\Delta^2 \left(\frac{M_s^2}{2\mu}\right)^2 \int d^2\ell \frac{3\ell_0^2 + \Delta^2 + \ell_\parallel^2}{(\ell_0^2 - \ell_\parallel^2 - \Delta^2)^3} \;. \tag{38}$$

The $\mathcal{K}$ is canceled by the tadpole $\mathcal{N}$ if $M_s = 0$ as it should, since for $M_s = 0$ the excitation has to be a true Goldstone boson; adding the π_5 contribution the one loop effective Lagrangian at zero momentum is therefore



$$= \frac{\pi_4^2 + \pi_5^2}{2}\frac{\mu^2}{\pi^2 F^2}\frac{21 - 8\log 2}{36}\left(\frac{M_s^2}{2\mu}\right)^2 \equiv \mu_4^2 K^+ K^- \;, \text{c.d.d.} \tag{39}$$

where we have used the expression of F in the CFL phase, appropriated since we are working at the leading order in

the strange quark mass [19, 39, 40],

$$F^2(M_s=0)=\frac{\mu^2}{\pi^2}\frac{21-8\log 2}{36}\ . \tag{40}$$

Eq. (39) is in agreement with [19, 42]; to our knowledge it is the first time that this important result is deduced by a one loop calculation of the effective action.

Before going on we comment on the choice of the chemical potentials. We have repeated the above calculation setting μ_8 to an arbitrary value, and for simplicity leaving $\mu_e=\mu_3=0$. The loop expressions in this case are complicated and therefore we do not show them, but the final result is very simple, namely:

$$\mathcal{L}(p=0)=-\mu_8\frac{M_s^2}{2\mu}\left(K^+K^-+K^0\bar{K}^0\right)\ . \tag{41}$$

We notice that the result is strongly dependent on the choice of μ_8. We obtain the correct result (39), that is the squared mass consistent with the effective lagrangian approach [19, 44, 45, 46], if and only if we set $\mu_8=-M_s^2/2\mu$, its value in the neutral CFL phase. As stressed in the introduction, this result is not surprising: as a matter of fact, it has been shown in [44, 46] that in order to properly write the effective lagrangian of the neutral CFL phase in terms of the colorless pion fields, one has to get rid of the colored components, and this is achieved if and only if the neutrality conditions are properly implemented. Thus, in order to reproduce the result of the effective lagrangian from the microscopic calculation, one has to chose the chemical potentials in order to fulfill the neutrality conditions.

As explained in the previous section, when one computes the meson masses, one has to add to the lagrangian in Eq. (31) the shift of the vacuum energy due to finite quark masses, related in the high effective theory to operators of order $1/\mu^2$. In the CFL phase with massless quark propagators the result of this calculation is [39, 41, 42]

$$m^2_{K^\pm}=\frac{4A}{F^2}M_d(M_u+M_s)\ , \tag{42}$$

$$m^2_{K^0/\bar{K}^0}=\frac{4A}{F^2}M_u(M_s+M_d)\ , \tag{43}$$

with $A=\Delta\bar{\Delta}/2\pi^2\log(\mu/\Delta)$ in a NJL model (in QCD $A=3\Delta^2/4\pi^2$). As already explained in the previous section, these results are not modified by the finite mass and the color chemical potential in the quark loops. Moreover we should notice that such kind of corrections vanish in the limit $M_{u,d}\to 0$, which is the limit we are taking since in the neutrality conditions in use we do not consider the effect of the light quarks; on the other hand, the effective chemical potential terms (32), (33) survive in this limiting case. For these reasons we do no longer discuss the contribution m_K arising from to the anti-gap action, using for them the leading order results (42), (43). As for the charged pions and the unflavored meson fields, at this order they are massless.

The complete lagrangian of the kaon modes at $p=0$ is thus given by

$$\mathcal{L}(p=0)=(\mu_4^2-m^2_{K^\pm})K^+K_-+(\mu_6^2-m^2_{K^0})K^0\bar{K}_0\ . \tag{44}$$

The terms (32), (33) are usually referred as effective chemical potentials, as they enter into the meson lagrangian via a covariant derivative as a typical meson chemical potential does,

$$\mathcal{L}\sim[(\partial_0+i\mu_4)K^+][(\partial_0-i\mu_4)K^-]\ . \tag{45}$$

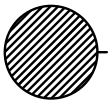
Since they give rise to a mass term with the wrong sign, when $\mu_4>m_K$ the ground state is unstable toward the formation of a kaon condensate [19]. In our approximation scheme, $m_K\simeq 0$ and $\mu_4^2>0$, thus the kaon condensation occurs for each value of $M_s\neq 0$.

B. Numerical results

The above results (32), (33) are confirmed by a numerical evaluation of the one loop diagrams (29), (30) for values of M_s^2/μ up to 2Δ; above this critical value the transition to gCFL phase occurs and one has to consider the contribution of the gapless modes in the quark loops, together with the conditions $\mu_e\neq 0$ and $\mu_3\neq 0$, which is beyond the scope of this paper.

The result of such a calculation is shown in Fig. 2, where we plot F^2 times the squared mass of the kaon modes $M^2\equiv-\mu_4^2$, obtained by the numerical evaluation of the loops in the case $M_u=M_d=0$ (in this case the kaon mass coincides with the minus of the effective chemical potential μ_4^2 since $m_K=0$). In the figure, diamonds and stars correspond respectively to $\Delta=25$ MeV and $\Delta=75$ MeV; the squares correspond to the leading order solution, which does not depend on Δ. The dependence on the gap parameter Δ when one approaches the value $M_s^2/2\mu\approx\Delta$ can be

easily understood by looking at the next-to-leading order correction to the result quoted in Eq. (39). We find

$$F^2 \times$$  $$= K^+K^- \left(\frac{M_s^2}{2\mu}\right)^2 \left[c_0 + c_2 \left(\frac{M_s^2}{2\mu\Delta}\right)^2\right] , \qquad (46)$$

where $c_0 = (21 - 8\log 2)\mu^2/36\pi^2$ as can be read from Eq. (39) and $c_2 \approx 0.07$: the introduction of the higher order corrections introduces a dependence on Δ that vanishes only when $M_s^2/2\mu\Delta \to 0$.

We notice that at a given M_s, the inclusion of the higher order corrections increases the value of μ_4, the effect being more important for weak couplings. On the other hand, once $M_{u,d} \neq 0$, these corrections do not change m_K (42) in a significant way: as a matter of fact the relevant diagrams get the leading contribution from the hard region $\ell_\parallel \simeq \mu \gg \mu_8,\ M_s$ and thus do not depend on $\mu_8,\ M_s$. Therefore at a given M_s the higher order corrections have the net effect of increasing $\mu_4 - m_K$ with respect to the leading order result, thus favoring kaon condensation.

V. CONCLUSIONS

In this paper we have computed the effective chemical potentials of the $SU(3)_A$ Goldstone excitations in the neutral CFL phase of QCD, starting from a microscopic model of quarks interacting with the Goldstones. We have worked within the approximations $M_u = M_d = 0$, thus retaining the neutrality conditions $\mu_e = \mu_3 = 0$, $\mu_8 \approx -M_s^2/2\mu$. Our results agree with those of Bedaque and Schafer [19], which are obtained by the authors on the basis of symmetry arguments and not starting from a microscopic model; therefore we offer here a diagrammatic derivation of their results, showing how they arise from loop effects.

In addition to the new derivation of the classical results, we have performed a numerical evaluation of the low energy parameters of the effective lagrangian of the meson modes, valid in the whole CFL domain $M_s^2/2\mu\Delta \leqslant 1$. Our results, summarized in Fig 2, show an enhancement of the kaon condensation with respect to the leading order result usually referred to in the literature.

We have performed the calculation in the color neutral state: this choice is motivated by gauge invariance, which requires that any homogeneous ground state of QCD has to be color neutral [44, 46]. Moreover, this is the only way to properly reproduce the low energy effective action of Ref. [19] starting from the microscopic theory. Indeed it has been shown [44, 46] that the requirement of color neutrality is equivalent to the removal, from the low energy effective lagrangian, of the color non-singlet fields. Translated to the microscopic calculation language, this is the same to say that the quark chemical potentials *must* be chosen in order to satisfy the color neutrality conditions. This is a necessary requirement of any calculation of the properties of the Goldstone bosons: any violation of color neutrality in the microscopic model is translated, in the effective theory, into the presence of non-singlet fields in the low-energy lagrangian. This reasoning is reinforced by the calculation leading to Eq. (41), where we have set μ_8 to an arbitrary value: we obtain the correct result, consistent with the low energy effective lagrangian, if and only if μ_8 is equal to its value in the neutral phase.

We have not computed both F and the velocity v of the meson fields. Their values in the neutral phase can be obtained from the results quoted in Refs. [28] once we notice that in the gapped CFL phase [39]

$$F_a^2 \propto m_{D,a}^2 , \qquad v_a^2 = m_{M,a}^2/m_{D,a}^2 , \qquad (47)$$

where F_a is the decay constant of the field π_a, and $m_{D,a}$ ($m_{M,a}$) correspond to the Debye (Meissner) screening mass of the gluon with adjoint index a.

It would be interesting to extend our computations to the case of $M_{u,d} \neq 0$, in order to reproduce the classical results of [19] for non vanishing light quark masses. To do that one has to consider the effect of the light quark masses to the neutrality conditions in order to make a consistent microscopic calculation. We expect that these effects are important in the region $M_s^2/\mu\Delta \ll 1$, becoming less and less important when one approaches the onset of the CFL→gCFL transition where $M_s \gg M_{u,d}$. Moreover, also retaining the approximation $M_u = M_d = 0$, it would be fine to increase the strange quark mass beyond the onset CFL→gCFL, in order to understand the role of the gapless fermion excitations on the spectrum of the axial excitations. From the formal point of view it is sufficient to introduce the proper values of the charge and color chemical potentials, but a preliminary analysis shows that the numerical work required for this interesting project is much more involved than the one presented here. Beside masses, the effective lagrangian parameters F and v should be evaluated in the gCFL regime: from the two flavor case we know that the squared velocities of some of the fields can be negative [31], and this would lead to the Goldstone currents studied in [32]. Finally, last but not least, it would be interesting to extend the calculations to finite temperature. Unfortunately this project is not trivial from the

numerical point of view as it requires the introduction of μ_e, μ_3 beside μ_8 [18]. We leave also this point to a future project.

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[1] H. Abuki, M. Kitazawa and T. Kunihiro, Phys. Lett. B **615**, 102 (2005) [arXiv:hep-ph/0412382]; H. Abuki and T. Kunihiro, Nucl. Phys. A **768**, 118 (2006) [arXiv:hep-ph/0509172].
[2] S. B. Ruster, V. Werth, M. Buballa, I. A. Shovkovy and D. H. Rischke, Phys. Rev. D **72**, 034004 (2005) [arXiv:hep-ph/0503184].
[3] D. Blaschke, S. Fredriksson, H. Grigorian, A. M. Oztas and F. Sandin, Phys. Rev. D **72**, 065020 (2005) [arXiv:hep-ph/0503194].
[4] T. Hatsuda, M. Tachibana, N. Yamamoto and G. Baym, Phys. Rev. Lett. **97**, 122001 (2006) [arXiv:hep-ph/0605018].
[5] N. Yamamoto, M. Tachibana, T. Hatsuda and G. Baym, arXiv:0704.2654 [hep-ph].
[6] K. Rajagopal and F. Wilczek, arXiv:hep-ph/0011333.
[7] M. G. Alford, Ann. Rev. Nucl. Part. Sci. **51**, 131 (2001) [arXiv:hep-ph/0102047].
[8] G. Nardulli, Riv. Nuovo Cim. **25N3**, 1 (2002) [arXiv:hep-ph/0202037].
[9] S. Reddy, Acta Phys. Polon. B **33**, 4101 (2002) [arXiv:nucl-th/0211045].
[10] T. Schafer, arXiv:hep-ph/0304281.
[11] D. H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004) [arXiv:nucl-th/0305030].
[12] M. Buballa, Phys. Rept. **407**, 205 (2005) [arXiv:hep-ph/0402234].
[13] M. Huang, Int. J. Mod. Phys. E **14**, 675 (2005) [arXiv:hep-ph/0409167].
[14] I. A. Shovkovy, Found. Phys. **35**, 1309 (2005) [arXiv:nucl-th/0410091].
[15] M. Alford and K. Rajagopal, arXiv:hep-ph/0606157.
[16] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B **537**, 443 (1999) [arXiv:hep-ph/9804403].
[17] N. J. Evans, J. Hormuzdiar, S. D. H. Hsu and M. Schwetz, Nucl. Phys. B **581**, 391 (2000) [arXiv:hep-ph/9910313].
[18] M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. Lett. **92**, 222001 (2004) [arXiv:hep-ph/0311286]; M. Alford, C. Kouvaris and K. Rajagopal, Phys. Rev. D **71**, 054009 (2005) [arXiv:hep-ph/0406137]; K. Fukushima, C. Kouvaris and K. Rajagopal, Phys. Rev. D **71**, 034002 (2005) [arXiv:hep-ph/0408322].
[19] P. F. Bedaque and T. Schafer, Nucl. Phys. A **697**, 802 (2002) [arXiv:hep-ph/0105150]; P. F. Bedaque, Phys. Lett. B **524**, 137 (2002) [arXiv:nucl-th/0110049]; T. Schafer, Phys. Rev. Lett. **85**, 5531 (2000) [arXiv:nucl-th/0007021].
[20] D. B. Kaplan and S. Reddy, Phys. Rev. D **65**, 054042 (2002) [arXiv:hep-ph/0107265].
[21] R. Casalbuoni, R. Gatto, N. Ippolito, G. Nardulli and M. Ruggieri, Phys. Lett. B **627**, 89 (2005) [Erratum-ibid. B **634**, 565 (2006)] [arXiv:hep-ph/0507247]; M. Mannarelli, K. Rajagopal and R. Sharma, Phys. Rev. D **73**, 114012 (2006) [arXiv:hep-ph/0603076]; K. Rajagopal and R. Sharma, Phys. Rev. D **74**, 094019 (2006) [arXiv:hep-ph/0605316]; N. D. Ippolito, G. Nardulli and M. Ruggieri, JHEP **0704**, 036 (2007) [arXiv:hep-ph/0701113].
[22] I. Shovkovy and M. Huang, Phys. Lett. B **564**, 205 (2003) [arXiv:hep-ph/0302142]; M. Huang and I. Shovkovy, Nucl. Phys. A **729**, 835 (2003) [arXiv:hep-ph/0307273].
[23] E. Gubankova, W. V. Liu and F. Wilczek, Phys. Rev. Lett. **91**, 032001 (2003) [arXiv:hep-ph/0304016].
[24] M. G. Alford, J. A. Bowers and K. Rajagopal, Phys. Rev. D **63**, 074016 (2001) [arXiv:hep-ph/0008208]; J. A. Bowers and K. Rajagopal, Phys. Rev. D **66**, 065002 (2002) [arXiv:hep-ph/0204079]; R. Casalbuoni, M. Ciminale, M. Mannarelli, G. Nardulli, M. Ruggieri and R. Gatto, Phys. Rev. D **70**, 054004 (2004) [arXiv:hep-ph/0404090].
[25] T. Schafer, Phys. Rev. D **62**, 094007 (2000) [arXiv:hep-ph/0006034]; M. G. Alford, J. A. Bowers, J. M. Cheyne and G. A. Cowan, Phys. Rev. D **67**, 054018 (2003) [arXiv:hep-ph/0210106]; A. Schmitt, Phys. Rev. D **71**, 054016 (2005) [arXiv:nucl-th/0412033].
[26] D. N. Aguilera, D. Blaschke, M. Buballa and V. L. Yudichev, Phys. Rev. D **72**, 034008 (2005) [arXiv:hep-ph/0503288]; F. Marhauser, D. Nickel, M. Buballa and J. Wambach, Phys. Rev. D **75**, 054022 (2007) [arXiv:hep-ph/0612027].
[27] M. Huang and I. A. Shovkovy, Phys. Rev. D **70**, 051501 (2004) [arXiv:hep-ph/0407049]; M. Huang and I. A. Shovkovy, Phys. Rev. D **70**, 094030 (2004) [arXiv:hep-ph/0408268].
[28] R. Casalbuoni, R. Gatto, M. Mannarelli, G. Nardulli and M. Ruggieri, Phys. Lett. B **605**, 362 (2005) [Erratum-ibid. B **615**, 297 (2005)] [arXiv:hep-ph/0410401]; M. Alford and Q. h. Wang, J. Phys. G **31**, 719 (2005) [arXiv:hep-ph/0501078]; K. Fukushima, Phys. Rev. D **72**, 074002 (2005) [arXiv:hep-ph/0506080].
[29] E. V. Gorbar, M. Hashimoto and V. A. Miransky, Phys. Lett. B **632**, 305 (2006) [arXiv:hep-ph/0507303]; E. V. Gorbar, M. Hashimoto and V. A. Miransky, Phys. Rev. D **75**, 085012 (2007) [arXiv:hep-ph/0701211]; M. Hashimoto and V. A. Miransky, arXiv:0705.2399 [hep-ph].
[30] M. Huang, Phys. Rev. D **73**, 045007 (2006) [arXiv:hep-ph/0504235].
[31] R. Gatto and M. Ruggieri, arXiv:hep-ph/0703276.
[32] T. Schafer, Phys. Rev. Lett. **96**, 012305 (2006) [arXiv:hep-ph/0508190]; A. Gerhold and T. Schafer,

Phys. Rev. D **73**, 125022 (2006) [arXiv:hep-ph/0603257]; A. Gerhold, T. Schafer and A. Kryjevski, Phys. Rev. D **75**, 054012 (2007) [arXiv:hep-ph/0612181].

[33] I. Giannakis and H. C. Ren, Phys. Lett. B **611**, 137 (2005) [arXiv:hep-ph/0412015];

[34] I. Giannakis and H. C. Ren, Nucl. Phys. B **723**, 255 (2005) [arXiv:hep-th/0504053]; I. Giannakis, D. Hou, M. Huang and H. c. Ren, Phys. Rev. D **75**, 011501 (2007) [arXiv:hep-ph/0606178]; M. Ciminale, G. Nardulli, M. Ruggieri and R. Gatto, Phys. Lett. B **636**, 317 (2006) [arXiv:hep-ph/0602180].

[35] D. K. Hong, Phys. Lett. B **473**, 118 (2000) [arXiv:hep-ph/9812510]; D. K. Hong, Nucl. Phys. B **582**, 451 (2000) [arXiv:hep-ph/9905523].

[36] T. Schafer, Nucl. Phys. A **728**, 251 (2003) [arXiv:hep-ph/0307074].

[37] R. Casalbuoni and R. Gatto, Phys. Lett. B **464**, 111 (1999) [arXiv:hep-ph/9908227].

[38] D. B. Kaplan and S. Reddy, Phys. Rev. Lett. **88**, 132302 (2002) [arXiv:hep-ph/0109256]; P. Jaikumar, M. Prakash and T. Schafer, Phys. Rev. D **66**, 063003 (2002) [arXiv:astro-ph/0203088]; M. G. Alford, M. Braby, S. Reddy and T. Schafer, arXiv:nucl-th/0701067.

[39] D. T. Son and M. A. Stephanov, Phys. Rev. D **61**, 074012 (2000) [arXiv:hep-ph/9910491]; D. T. Son and M. A. Stephanov, Phys. Rev. D **62**, 059902 (2000) [arXiv:hep-ph/0004095].

[40] R. Casalbuoni, R. Gatto and G. Nardulli, Phys. Lett. B **498**, 179 (2001) [Erratum-ibid. B **517**, 483 (2001)] [arXiv:hep-ph/0010321].

[41] S. R. Beane, P. F. Bedaque and M. J. Savage, Phys. Lett. B **483**, 131 (2000) [arXiv:hep-ph/0002209].

[42] T. Schafer, Phys. Rev. D **65**, 074006 (2002) [arXiv:hep-ph/0109052].

[43] C. Manuel and M. H. G. Tytgat, Phys. Lett. B **479**, 190 (2000) [arXiv:hep-ph/0001095].

[44] A. Kryjevski, Phys. Rev. D **68**, 074008 (2003) [arXiv:hep-ph/0305173].

[45] M. Buballa, Phys. Lett. B **609**, 57 (2005) [arXiv:hep-ph/0410397].

[46] M. M. Forbes, Phys. Rev. D **72**, 094032 (2005) [arXiv:hep-ph/0411001].

[47] D. Ebert and K. G. Klimenko, Phys. Rev. D **75**, 045005 (2007) [arXiv:hep-ph/0611385]; D. Ebert, K. G. Klimenko and V. L. Yudichev, arXiv:0705.2666 [hep-ph].

[48] T. Eguchi, Phys. Rev. D **14**, 2755 (1976).

[49] R. Anglani, R. Gatto, N. D. Ippolito, G. Nardulli and M. Ruggieri, arXiv:0706.1781 [hep-ph].

[50] M. Alford and K. Rajagopal, JHEP **0206**, 031 (2002) [arXiv:hep-ph/0204001].

[51] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); Y. Nambu and G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961).

[52] S. P. Klevansky, Rev. Mod. Phys. **64** (1992) 649.

[53] T. Hatsuda and T. Kunihiro, Phys. Rept. **247**, 221 (1994) [arXiv:hep-ph/9401310].

[54] M. Buballa and I. A. Shovkovy, Phys. Rev. D **72**, 097501 (2005) [arXiv:hep-ph/0508197].

[55] R. Casalbuoni, F. De Fazio, R. Gatto, G. Nardulli and M. Ruggieri, Phys. Lett. B **547**, 229 (2002) [arXiv:hep-ph/0209105].

[56] A. W. Steiner, S. Reddy and M. Prakash, Phys. Rev. D **66**, 094007 (2002) [arXiv:hep-ph/0205201].

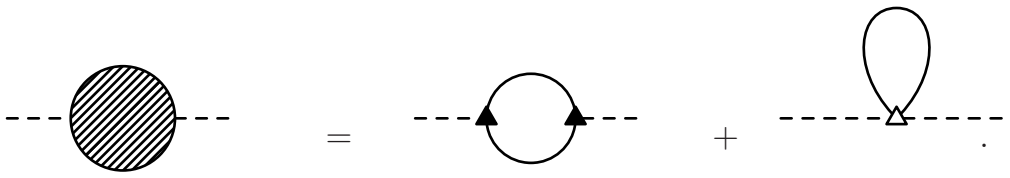

FIG. 1: One loop effective action of the "pion" modes π_a. External dashed lines denote the meson field, while solid lines stem for the quark propagators. Black triangles are for K_3 vertices, while empty triangles are for K_4 vertices.

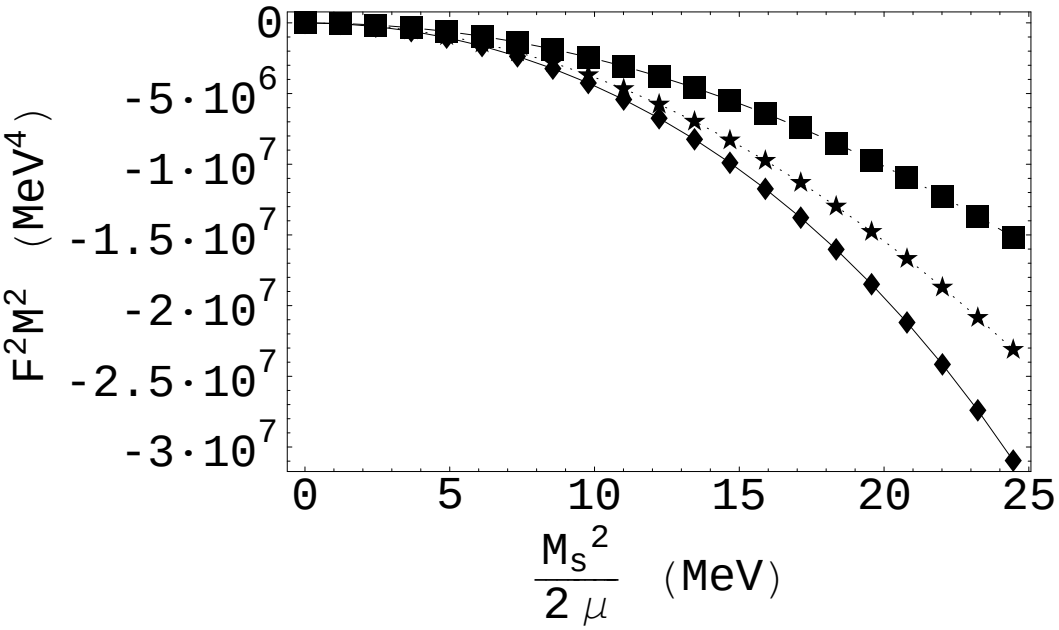

FIG. 2: Squared mass of the kaon modes times F^2, in the neutral CFL phase with massless u and d quarks, against $M_s^2/2\mu$. Diamonds and stars correspond respectively to $\Delta = 25$ MeV and $\Delta = 75$ MeV. Squares correspond to the leading order solution, which does not depend on Δ. In the figure we have chosen $\mu = 500$ MeV as a reference value of the baryon chemical potential.